COMMUNICATION ENGINEERING

Joint and Conditional Entropy

PRELIMINARY CONCEPTS

- Joint Probability of X and Y is p(X,Y), probability that X and *Y occur* simultaneously
  - If *X*, Y are independent,

$$p(X, Y) = P(X)P(Y)$$

- Conditional probability of *X* given *Y*, *P*(*X*|*Y*), is probability that *X* takes on a particular value given *Y* has a particular value

- Conditional probability of *Y* given X *P*(*Y*|*X*), is probability that *X* takes on a particular value given *Y* has a particular value

Relationship (Bayes Theorem)

- $P(X, Y) = P(X \mid Y)\, P(Y) = P(X)\, P(Y \mid X)$

  or

  $P(X \mid Y) = P(X, Y) / P(Y)$

  $P(Y \mid X) = P(X,Y) / P(X)$

- if X, Y independent:
  - $P(X|Y) = P(X)$
  - $P(Y|X)=P(Y)$

JOINT ENTROPY

- H(X,Y)- Joint Entropy of X and Y(average uncertainty of the communication system as a whole)

$$H(X,Y) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k)\log_2 p(x_j, y_k)$$

- H(X) - Entropy of the transmitter /Average uncertainty of the channel input

Where

$$H(X) = -\sum_{j=1}^{m} p(x_j)\log_2 p(x_j)$$

$$p(x_j) = \sum_{k=1}^{n} p(x_j, y_k)$$

- H(Y) - Entropy of the receiver / Average uncertainty of the channel output

$$H(Y) = -\sum_{k=1}^{n} p(y_k)\log_2 p(y_k)$$

Where

$$p(y_k) = \sum_{j=1}^{m} p(x_j, y_k)$$

CONDITIONAL ENTROPY

- H(Y|X): average uncertainty about the channel output given that X was transmitted
  - It indicates how well one can recover the received symbols from the transmitted symbols
  - It gives a measure of error or noise

$$H(Y|X) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k)\log_2 p(y_k | x_j)$$

- H(X|Y): average uncertainty about the channel input after the channel output has been observed
  - It indicates how well one can recover the transmitted symbols from the received symbols
  - It gives a measure of equivocation

$$H(X|Y) = -\sum_{j=1}^{m}\sum_{k=1}^{n} p(x_j, y_k)\log_2 p(x_j | y_k)$$

RELATIONSHIPS BETWEEN JOINT AND CONDITIONAL ENTROPIES

- $H(X,Y) = H(X|Y) + H(Y)$

  or

  $H(X|Y) = H(X,Y) - H(Y)$

- $H(X,Y) = H(Y|X) + H(X)$

  or

  $H(Y|X) = H(X,Y) - H(X)$

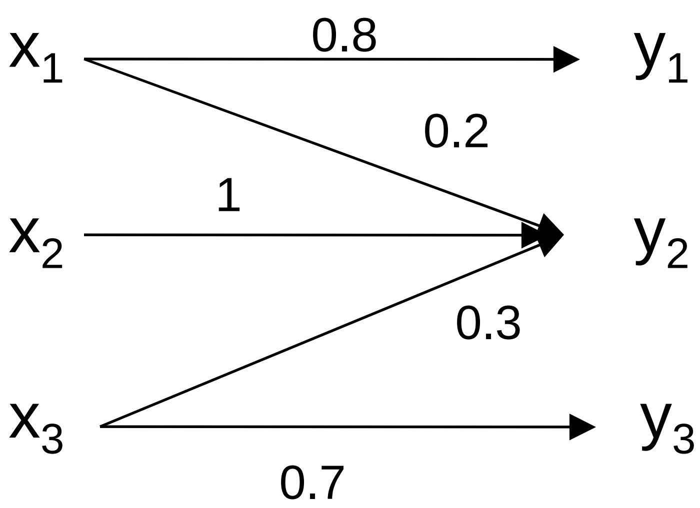
- When X and Y are independent

  $H(Y|X) = H(Y)$

  and $H(X|Y) = H(X)$

PROBLEMS

- A discrete source transmits messages x_1 , x_2 , x_3 with probabilities 0.3,0.4 and 0.4 The source is connected to the channel given in figure. Calculate all the associated entropies



THANK YOU